Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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6	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MM05

Unit Mechanics 5

Friday 21 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 A simple pendulum consists of a small sphere attached to one end of a light inextensible string of length 2 metres. The other end of the string is attached to a fixed point. The pendulum is set into motion in a vertical plane.

(a) Find the period of the motion for small oscillations. *(2 marks)*

(b) During the motion, the maximum angle between the string and the vertical is $\frac{\pi}{20}$. Find the maximum speed of the sphere on the end of the string. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 1

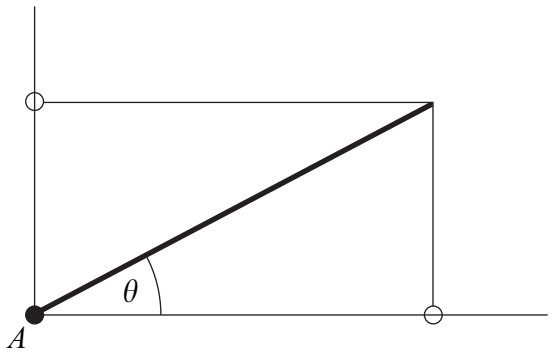
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QUESTION PART REFERENCE

Answer space for question 1

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2 Two light elastic strings both have modulus of elasticity $2mg$ and natural length a. The strings are attached to the end of a uniform rod of mass m and length $2a$. The other ends of the strings are attached to small light rings that slide on smooth fixed horizontal and vertical rods, with one string horizontal and the other vertical, as shown in the diagram. The other end of the rod is smoothly hinged at A.



The rod is at an angle θ to the horizontal, where $\frac{\pi}{6} < \theta < \frac{\pi}{3}$.

The gravitational potential energy is taken to be zero at the level of A.

(a) Show that the potential energy of the system, V, is given by

$$V = mga(6 - 4\cos\theta - 3\sin\theta)$$ *(5 marks)*

(b) Given that the system is in equilibrium, find θ. *(4 marks)*

(c) Show that the value found in part **(b)** gives a position of stable equilibrium. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 2

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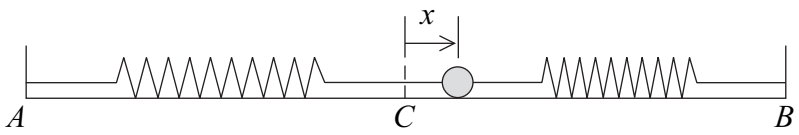
QUESTION PART REFERENCE

Answer space for question 2

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3 A particle, of mass 9 kg, is attached to two identical springs. The other ends of the springs are attached to fixed points, A and B, which are 1.2 metres apart on a smooth horizontal surface. The springs have modulus of elasticity 45 N and natural length 0.4 m.

The particle is released from rest at a distance of 0.5 metres from B and moves on the line AB. The midpoint of AB is C. At time t seconds after release, the displacement of the particle from C is x metres, where the direction from A to B is taken to be positive.



(a) Show that the resultant force on the particle, at time t, is $-225x$ newtons. *(4 marks)*

(b) Hence show that the particle moves with simple harmonic motion. *(2 marks)*

(c) State the period of this motion. *(2 marks)*

(d) Find the speed of the particle when it is 0.05 metres from C. *(4 marks)*

(e) Write down an expression for x in terms of t. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

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QUESTION PART REFERENCE

Answer space for question 3

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4 A particle moves so that, at time t, its polar coordinates (r, θ) are given by

$$r = 1 + 2\cos\theta \quad \text{and} \quad \theta = 4t$$

(a) Show that the speed, v, of the particle is given by

$$v = k\sqrt{5 + 4\cos\theta}$$

where k is an integer. *(5 marks)*

(b) Find the magnitude of the transverse component of the acceleration when the radial component of the acceleration is zero. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 4

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QUESTION PART REFERENCE

Answer space for question 4

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5 **In this question, take $g = 10\,\text{m s}^{-2}$.**

A bungee jumper has mass 75 kg. He uses an elastic rope of natural length 12 m and modulus of elasticity 450 N. He falls vertically, and when the rope becomes taut for the first time, he is travelling at $12.5\,\text{m s}^{-1}$. Assume that, once the rope is taut, the bungee jumper experiences an air resistance force that has magnitude $15v$ newtons, where $v\,\text{m s}^{-1}$ is his speed.

At time t seconds after the rope has become taut, the extension of the rope is x metres.

(a) Show that

$$10\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 100$$ *(4 marks)*

(b) Find x in terms of t. *(10 marks)*

(c) Find the value of t when the bungee jumper first comes instantaneously to rest. *(5 marks)*

QUESTION PART REFERENCE

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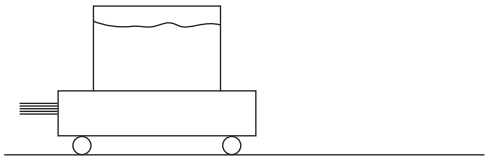
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6 An inventor creates a vehicle that is propelled by pumping water out of the back of the vehicle. The water is expelled from the back of the vehicle by a pump mounted inside the vehicle. The water is expelled horizontally at a speed of $U\,\mathrm{m\,s^{-1}}$ relative to the vehicle. The total mass of the vehicle and the water in the vehicle is $m\,\mathrm{kg}$ at time t seconds after the water starts to be pumped out of the vehicle. The velocity of the vehicle at time t is $v\,\mathrm{m\,s^{-1}}$.

Assume that no resistance forces act on the vehicle.



(a) Show that, while the water is being expelled from the vehicle,

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = -U\frac{\mathrm{d}m}{\mathrm{d}t}$$ *(4 marks)*

(b) The mass of the vehicle, excluding the water, is $M\,\mathrm{kg}$. Initially, the vehicle is at rest and it contains $M\,\mathrm{kg}$ of water. The water leaves the vehicle at a constant rate of $\lambda\,\mathrm{kg}$ per second.

(i) Find v in terms of M, U, λ and t. *(7 marks)*

(ii) Find, in terms of U, the speed of the vehicle when it runs out of water. *(3 marks)*

QUESTION PART REFERENCE

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END OF QUESTIONS

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